The principle of reflection via nested sequents

Abstract

In Sambin, Battilotti and Faggian (2000), they discuss a natural way of introducing logical constants in a proof-theoretical method, which can belong to the area of proof-theoretic semantics broadly considered. Roughly, the idea is 1) to use an equivalence formula as a "definition" of a logical constant, 2) to derive ordinary operational rules in a cut-free sequent calculus for the logical constant only via minimal background assumptions, and 3) to understand various non-classical logics as structural variants. The idea works well for multiplicative and additive conjunction and disjunction. However, it turns out that handling implication is not entirely unproblematic.

In this talk, we propose that in order to naturally introduce implication, we should use a generalized proof-theoretic framework, in particular what are called "nested sequents," keeping Sambin et al.'s conceptual motivations as much as possible. Nested sequents have already been used in modal logic as a natural proof-theoretic representation of Kripke semantics. However, our discussions can give another motivation for nested sequents that is unrelated to Kripke semantics.

We argue that such a generalization is motivated both technically and philosophically. From a philosophical point of view, we first point out that there are some potential conceptual problems in intuitionistic logic and then we argue that, in order to analyze the problems, we may be motivated in formulating strictly weaker logics than intuitionistic logic. When we do so, there are several reasons why traditional sequent calculi are not particularly convenient. From a technical point of view, we show that nested sequents can formulate a variety of non-classical logics in a uniform way. (We present both our own results and a survey of the literature.)

Time permitting, we also discuss the issue of how nested sequents can be compared with other generalizations of sequent calculi.